Translate the following sentences into the language of sentential logic using the abbreviations given to you.

O = “Oxygen is available.”
F = “Fuel is available.”
L = “We can light the fire.”
D = “The fire will die out soon.”

1. “Oxygen and fuel are both unavailable.”
   \( \sim O \& \sim F \)

2. “We can light the fire if fuel is available.”
   \( F \supset L \)

3. “The fire will die out soon unless oxygen is available.”
   \( \sim O \supset D \)

4. “We can light the fire, but the fire will die out soon if either fuel or oxygen is unavailable.”
   \( L \& ((\sim F \lor \sim O) \supset D) \)

5. “The fire will die out soon unless both oxygen and fuel are available.”
   \( \sim (O \& F) \supset D \)

6. “Either fuel and oxygen are available, or the fire will die out soon.”
   \( (F \& O) \lor D \)

C = “Gina is capable of flying the plane.”
S = “Gina should be flying the plane.”
D = “Gina is drunk.”
E = “Ed should be monitoring the radar.”

7. “If Gina is drunk, she shouldn’t be flying the plane.”
   \( D \supset \sim S \)

8. “Gina shouldn’t fly the plane if she isn’t capable of it.”
   \( \sim C \supset \sim S \)

9. “Gina should be flying the plane, and Ed should be monitoring the radar.”
   \( S \& E \)

10. “Unless Gina is drunk and incapable of flying the plane, Ed should be monitoring the radar.”
    \( \sim (D \& \sim C) \supset E \)
Translate the following sentences into the language of quantifier logic using the given abbreviations. Remember that you do not need to worry about tense.

\[ W_x = x \text{ is wood.} \]
\[ D_x = x \text{ is damp.} \]
\[ F_x = x \text{ can catch on fire.} \]
\[ C_x = x \text{ is cedar.} \]
\[ B_x = x \text{ is burning.} \]
\[ t = \text{the torch} \]
\[ b = \text{the bonfire} \]

11. “The torch is damp, but it can still catch on fire if it’s a cedar torch.”
\[ D_t \land (C_t \supset F_t) \]

12. “The bonfire is the only thing burning.”
\[ \neg \exists x(x \neq b \land B_x) \text{ or equivalently } \forall x(B_x \supset x = b) \]

13. “The only kind of wood that can catch on fire is cedar.”
\[ \neg \exists x((W_x \land F_x) \land \neg C_x) \text{ or equivalently } \forall x((W_x \land F_x) \supset C_x) \]

14. “Any wood that isn’t damp can catch on fire.”
\[ \forall x((W_x \land \neg D_x) \supset F_x) \]

15. “Not everything that can catch on fire is burning.”
\[ \neg \forall x(F_x \supset B_x) \]

16. “The torch can catch on fire only if it’s wooden.”
\[ F_t \supset W_t \]

17. “Any wood that isn’t cedar can catch on fire.”
\[ \forall x((W_x \land \neg C_x) \supset F_x) \]
Logic—Sample Final Examination E3 with Answers

Label each of the following sequences of symbols with a check mark if and only if it is a legitimate statement of logic. Mark the expression with an ‘X’ if and only if it is not a legitimate statement.

18. $\forall x \forall y (Q \supset K xy)$  X

19. $\sim Y \lor \forall x (Gab=x)$  X

20. $(z = y) \land P a$  X

21. $\exists xy (W x \supset F xy)$  X

22. $(\sim F \lor (\sim T \supset (W \land L)) \land \sim E)$  X

23. $\forall k \forall y (k \neq y \supset P k)$  X

Construct truth tables to test whether these arguments are valid or invalid. In the case of an invalid argument, indicate the row or rows that show that the argument is invalid by circling at least one of them.

24.  

<table>
<thead>
<tr>
<th>K</th>
<th>Q</th>
<th>L</th>
<th>~K &amp; Q</th>
<th>L ⊃ K</th>
<th>~L v ~Q</th>
</tr>
</thead>
<tbody>
<tr>
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<td>FT FT FT</td>
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</tbody>
</table>

Valid or invalid?  VALID  If it is invalid, circle any one row that proves that it is invalid.

25.  

<table>
<thead>
<tr>
<th>P</th>
<th>A</th>
<th>~P &amp; (~A v ~P)</th>
<th>P v A</th>
<th>A &amp; (~P ⊃ ~A)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>F</td>
<td>TF FT FT FT FT</td>
<td>FT T</td>
<td>FT FT FT FT</td>
</tr>
</tbody>
</table>

Valid or invalid?  INVALID  If it is invalid, circle any one row that proves that it is invalid.
Use the truth table method to determine whether the set of sentences is equivalent.

26. { "Bernie is not going if Wendy is”, “Wendy is going if Bernie isn’t” }

<table>
<thead>
<tr>
<th>W</th>
<th>B</th>
<th>W ⊃ ~B</th>
<th>~B ⊃ W</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
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<tr>
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<td>T</td>
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</tr>
<tr>
<td>F</td>
<td>F</td>
<td>TTTF</td>
<td>TFTF</td>
</tr>
</tbody>
</table>

Equivalent or inequivalent? INEQUIVALENT
If it is inequivalent, circle any one row that proves that it is inequivalent.

27. Use the truth table method to determine whether the set of sentences is consistent.
{ “Unless Cody’s fishing, he isn’t taking the truck,” “Cody is neither fishing nor taking the truck.” }

<table>
<thead>
<tr>
<th>F</th>
<th>T</th>
<th>~F ∨ ~T</th>
<th>~(F ∨ T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>FFTT</td>
<td>TTTT</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
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<td>FFTT</td>
</tr>
<tr>
<td>T</td>
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<td>TTTF</td>
<td>TTTF</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>TTTT</td>
<td>TFFT</td>
</tr>
</tbody>
</table>

Consistent or inconsistent? CONSISTENT
If it is consistent, circle any one row that proves that it is consistent.

For each of the following three sentences indicate whether it is a tautology, a contradiction, or a contingent sentence. Show some kind of formal proof. Use auxiliary premises if needed.

28. “Neither fish nor sharks inhabit the cave, but some white fish inhabit the cave.”

~(F ∨ S) & F

1. ~ (F ∨ S) & F
2. ~ (F ∨ S) 1, &
3. F 1, &
4. ~F 2, ~v
5. ~S 2, ~v

CONTRADICTION
29. “Helen isn’t here, and even if she is here, she isn’t.”

1. \( \neg H \land \neg H \)
2. \( \neg H \land 1, \& \)
3. \( \neg H \land 1, \& \)

1. \( \neg(\neg H \land \neg H) \)
2. \( \neg\neg H \neg\neg H \land 1, \& \)
3. \( H \land H \land 2, \neg \)

CONTESTANT

30. “Joe is a brother to all of his sister’s brothers.”

\[ \forall x(B_{xs} \supset B_{xj}) \]
\[ B_{js} \]
\[ \forall x \neg B_{xx} \]

\[ \sqrt{1. \forall x(B_{xs} \supset B_{xj})} \]
2. \( B_{js} \)
\[ \sqrt{3. \forall x \neg B_{xx}} \]
4. \( B_{js} \supset B_{xj} \land 1, \forall \)
5. \( \neg B_{js} \neg B_{xj} \land 4, \forall \)
6. \( x \neg B_{xj} \neg B_{xj} 3, \forall \)

CONTRADICTIO

31. Are the following sentences logically consistent? Show some kind of formal proof.
\{ A \land (B \supset C), \neg(C \supset B) \lor \neg A \}

\[ \sqrt{1. A \land (B \supset C)} \]
2. \( \neg(C \supset B) \lor \neg A \land 1, \& \)
3. \( A \land 1, \& \)
4. \( B \supset C \land 1, \& \)
5. \( \neg(C \supset B) \neg A \land 2, \lor \)
6. \( C \lor 5, \neg \)
7. \( \neg B \lor 5, \neg \)
8. \( \neg B \lor C \lor 4, \lor \)

Consistent
Use the truth tree method to determine whether the set of sentences is consistent.

32. \{ R \lor Q, \ P \Rightarrow R, \ \neg (Q \lor P) \} 

There is an error in the above tree. The negation in front of the R in line 7 should not be there.

Use the truth tree method to determine whether the argument is valid.

33. \begin{align*}
J & \supset (M \lor \neg M) \\
\neg Q & \supset (M \land \neg M) \\
\neg J & \land Q
\end{align*} 

\begin{align*}
1. \ J \supset (M \lor \neg M) \\
2. \ \neg Q \supset (M \land \neg M) \\
3. \ \neg (J \land Q) \\
4. \ \neg J & \supset (M \lor \neg M) 1, 2 \\
5. \ \neg M & \lor \neg ^{\, \, 4} \\
6. \ \neg ^{\, \, 5} & \lor \neg ^{\, \, 4} \\
7. \ \neg ^{\, \, 6} & Q \land \neg M 2, 3 \\
8. \ \neg ^{\, \, 6} & M 7, 8 \\
9. \ \neg ^{\, \, 6} & x 7, 8 \\
10. \ \neg ^{\, \, 6} \lor \neg ^{\, \, 6} 3, 8
\end{align*} 

Valid
34. \( \forall x (Jx \supset Tx) \)

\( \forall x (\exists y (Jy \& Fxy) \supset \exists z (Tz \& Fxz)) \)

\[ \begin{align*}
1. \forall x (Jx & \supset Tx) \\
2. & \forall x [\exists y (Jy \& Fxy) \supset \exists (T'_{y} \& F'^{2}xy)] \\
3. & \exists x [\exists y (Jy \& Fxy) \supset \exists (T'_{y} \& F'^{2}xy)] \\
4. & \exists (T'_{y} \& F'^{2}xy) \\
5. & \exists (T'_{y} \& F'^{2}xy) \\
6. & \exists (T'_{y} \& F'^{2}xy) \\
7. & J_{b} \& F'^{2}ab \\
8. & J_{b} \\
9. & F'^{2}ab \\
10. & J_{b} \supset (T'_{b} \& F'^{2}ab) \\
11. & \sim (T'_{b} \& F'^{2}ab) \\
12. & \sim T'_{b} \sim F'^{2}ab \\
13. & J_{b} \supset T'_{b} \\
14. & \sim T'_{b} \sim x \\
\end{align*} \]

Valid
Use the **truth tree** method to determine whether the set of sentences is consistent.

35. \( \{ \forall x \exists y (Kxy) \lor L, \neg \exists y (Kby) \lor L \supset \forall x (\exists z (Kbz)) \} \)

36. Is there any sentence that is both a contradiction and a tautology? Explain.

No. By definition a contradiction is false in every possible world and a tautology is true in every possible world. So, the only way they could be consistent is if there are no possible worlds. But there is a possible world, the actual world.

37. The menu at the cafeteria says that the roast mutton dish comes with your choice of three vegetable side dishes. Jackie reads the menu, looks at the available vegetables, and says to her buddy Ingred, I can have green beans or mashed potatoes. Ingred, trying to remember from her logic class last semester how to formalize Jackie’s statement, thinks to herself that Jackie is claiming \((G \& P)\), where \(G = \text{“Jackie can have green beans,”}\) and \(P = \text{“Jackie can have mashed potatoes.”}\) Which of the following is true?

a) Ingred has made an error. She should have used ‘\(\lor\)’ in her translation, not ‘\(\&\)’.
b) Jackie said something logically invalid. She should have used ‘and’ instead of ‘or’.
c) This situation shows that logic doesn’t always work in the real world.
d) **This example illustrates that sometimes ‘or’ should be translated as ‘&’.**
e) Ingred never should have broken up Jackie’s statement into a conjunction because Jackie’s disjunction came within the scope of the word ‘can’ which denotes possibility.