Translate the following sentences into the language of sentential logic using the abbreviations given to you.

B = “The forest contains beech trees.”
C = “The forest will be cleared.”
E = “The forest contains elm trees.”
O = “The forest is replanted with oaks.”

1. “The forest has beeches and elms.”
B & E

2. “The forest will be cleared if it doesn’t contain any elms or beeches.”
~(E v B) ⊃ C

3. “Either elm trees and beech trees are in the forest, or the forest will be cleared.”
(E & B) v C

4. “The forest doesn’t contain both elm trees and beech trees.”
~(E & B)

5. Unless there are elms in the forest, it will be cleared and replanted with oaks.”
~E ⊃ (C & O)

6. “If the forest isn’t cleared, it won’t be replanted with oaks.”
~C ⊃ ~O

L = “Our leader is at the meeting.”
A = “We will be able to accomplish something.”
P = “The protesters are allowed near the premises.”
E = “The press is allowed near the premises.”

7. “Without our leader being at the meeting, we won’t be able to get anything accomplished.”
~L ⊃ ~A

8. “We’ll be able to accomplish something only if the protesters are not allowed near the premises.”
A ⊃ ~P

9. “The protestors won’t be allowed near the premises if our leader is at the meeting.”
L ⊃ ~P

10. “If neither the protestors nor press are allowed near the premises, we will be able to
accomplish something.”
\((P \lor E) \supset A\)

Translate the following sentences into the language of quantifier logic using the abbreviations given to you.

\(B_{xy} = x\) is a brother to \(y\)

\(S_{xy} = x\) is a sister to \(y\)

\(I_{xy} = x\) is in \(y\)

\(C_x = x\) is a car

\(O_{xy} = x\) owns \(y\)

\(D_x = x\) is a dress

\(W_x = x\) is white

\(c = Christine\)

\(a = Amber\)

\(s = Steve\)

11. “Amber doesn’t own a white dress.”
\(\neg \exists y (W_y \land (D_y \land O_a y))\)

12. “Steve is not in Amber’s car.”
\(\neg \exists y (C_y \land (I_s y \land O_a y))\)

13. “Christine is an only child.”
\(\neg \exists y (B_y c \lor S_y c)\)

14. “Amber is Steve’s sister, but Christine isn’t.”
\(S_a s \land \neg S_c s\)

15. “One of Amber’s sisters is in Christine’s car.”
\(\exists x (S_a x \land \exists y (I_x y \land (C_y \land O_c y)))\)

16. “Christine has Amber’s dress in her car.”
\(\exists x ((D_x \land O_a x) \land \exists y (I_x y \land (C_y \land O_c y)))\)

17. “None of Steve’s brothers owns a white car.”
\(\neg \exists x (B_s x \land \exists y (C_y \land W_y \land O_x y))\)

18. “Some of Amber’s sister’s dresses are in her car.”
\(\exists x (D_x \land \exists y ((S_y \land O_y d) \land \exists z (I_x z \land (C_z \land O_a z))))\)

19. “Christine and Steve are currently located in a car together.”
\(\exists x (C_x \land (I_s x \land I_c x))\)
20. “Christine’s dress is in one of Steve’s cars.”
\[ \exists x((Dx \& Ox) \& \exists y(Ixy \& (Cy \& Osy))) \]

Label each of the following sequences of symbols with a check mark if and only if it is a legitimate statement of logic. Mark the expression with an ‘X’ if and only if it is not a legitimate statement. (1 point each)

21. \( W \lor (\sim T \supset \sim (W \& \sim T)) \& \sim T \)  
   \text{X}

22. \( \exists x \forall x(Cx \supset \sim xx) \)

(It is permissible to have \( \exists x \) on the outside even though there is already an \( x \) on the inside. Though, it is best not to do this in practice because it can lead to confusion.)

23. \( \exists x(Gh=x) \)
   \text{X}

24. \( \sim X(z = y) \& P \)
   \text{X}

25. \( \forall x \sim \forall y \sim (Q \supset Kxy) \)

26. \( \forall f(c \neq f \supset \sim I f) \)
   \text{X}

Construct truth tables to test whether these arguments are valid or invalid. In the case of an invalid argument, indicate the row or rows that show that the argument is invalid by circling at least one of them. (4 points.)

27. \[ \begin{array}{c|c|c|c|c|c|c} 
S & R & O & S \& \sim O & R \lor O & \sim R \supset \sim S \\
T & T & T & T & F & FT & FT \\
F & T & T & F & F & FT & FT \\
T & F & T & F & F & FT & FT \\
F & F & T & F & F & FT & FT \\
T & T & F & F & F & FT & FT \\
F & T & F & F & F & FT & FT \\
T & F & F & T & T & FT & TF \\
F & F & F & T & T & FT & TF \\
\end{array} \]

Valid or invalid? VALID  
If it is invalid, circle any one row that proves that it is invalid.
Logic—Sample Final Examination E2 with Answers

28. \(~E \lor (\neg H \land E)\)
   
   \(E \land \neg (H \supset \neg E)\)
   
   \(\neg (E \supset \neg (H \land E))\)

<table>
<thead>
<tr>
<th>E</th>
<th>H</th>
<th>(~E \lor (\neg H \land E))</th>
<th>E \land \neg (H \supset \neg E)</th>
<th>\neg (E \supset \neg (H \land E))</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>FT T FT F T</td>
<td>T T T F F T</td>
<td>T F F T T T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>TF T FT F F</td>
<td>F F F T T F</td>
<td>F T T T F F</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>FT T FT T T</td>
<td>T F F T F T</td>
<td>F T T T F T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>TF T TF F F</td>
<td>F F F T T F</td>
<td>F T T T F F</td>
</tr>
</tbody>
</table>

Valid or invalid? VALID  If it is invalid, circle any one row that proves that it is invalid.

Use the truth table method to determine whether the set of sentences is equivalent. (4 points)

29. \{ “Roger is not visiting if Doris is”, “Doris is visiting if Roger isn’t” \}

<table>
<thead>
<tr>
<th>R</th>
<th>D</th>
<th>D \supset \neg R</th>
<th>\neg R \supset D</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T F F T</td>
<td>F T T T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T T TF</td>
<td>TF T T</td>
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<tr>
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<td>F</td>
<td>F T FT</td>
<td>FT T F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>FT TF FT</td>
<td>TF F F</td>
</tr>
</tbody>
</table>

Equivalent or inequivalent? INEQUIVALENT  If it is inequivalent, circle any one row that proves that it is inequivalent.

30. Use the truth table method to determine whether the set of sentences is consistent. (4 points)
   \{ “Frank isn’t meeting the CFO unless Frank is golfing.”, “Frank is neither golfing nor meeting the CFO.” \}

<table>
<thead>
<tr>
<th>M</th>
<th>G</th>
<th>\neg G \supset \neg M</th>
<th>\neg (G \lor M)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>FT T FT</td>
<td>F T T T</td>
</tr>
<tr>
<td>F</td>
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<td>FT T TF</td>
<td>F T T F</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>TF F FT</td>
<td>F T T T</td>
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<tr>
<td>F</td>
<td>F</td>
<td>TF T TF</td>
<td>F T T F</td>
</tr>
</tbody>
</table>

Consistent or inconsistent? CONSISTENT  If it is consistent, circle any one row that proves that it is consistent.

For each of the following three sentences indicate whether it is a tautology, a contradiction, or a contingent sentence. Show some kind of formal proof. Use auxiliary premises if needed.

31. “Either cobras and vipers live here, or neither boas nor vipers live here.”
   \((C \land V) \lor \neg (B \lor V)\)
32. “Wendy is present only if she isn’t present.” (3 points)
\( W \supset \neg W \)

33. You need to add in several implicit premises to evaluate this sentence properly:
“Joe is a brother to his sister Angela, and Joe is a brother to all of Angela’s brothers.”
\( B_ja \land \forall x (B_{xa} \supset B_{jx}) \)
\( \forall x \sim B_{xx} \)

CONTRADICTION
34. Are the following sentences logically consistent? Show some kind of formal proof.
\{ \sim (Z \supset D) \lor \sim I, I \land (\sim D \supset \sim Z), \}
Use the truth tree method to determine whether the argument is valid.

35. \[(Y \lor \neg Y) \supset \neg N \]
   \[O \supset (U \land \neg U)\]
   \[\neg (N \lor O)\]

36. \[\neg \forall x (P_x \supset \exists w \neg D_{xw})\]
   \[\exists z (P_z \land \forall y D_{zy})\]
37. Are the following sentences logically equivalent to each other? Show some kind of formal proof.

“It’s not only people who are nice who help Jason.”
“Someone who isn’t nice is helping Jason.”

\[
\neg \forall x (P_x \supset (H_{xj} \supset \neg N_x)) \\
\exists x ((P_x \& \neg N_x) \& H_{xj})
\]
Logic—Sample Final Examination E2 with Answers

38. For each of the following sentences in the left hand column of the table below, indicate whether it is consistent with the statement \( P = \) “Not everyone is perfect.” Also indicate whether the sentence is entailed by \( P \).

<table>
<thead>
<tr>
<th>Sentence</th>
<th>Consistent with ( P )? (Y/N)</th>
<th>Entailed by ( P )? (Y/N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>“No one is perfect.”</td>
<td>Y</td>
<td>N</td>
</tr>
<tr>
<td>“Someone is perfect.”</td>
<td>Y</td>
<td>N</td>
</tr>
<tr>
<td>“No one is imperfect.”</td>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>“Not everyone is imperfect.”</td>
<td>Y</td>
<td>N</td>
</tr>
<tr>
<td>“Someone is imperfect.”</td>
<td>Y</td>
<td>Y</td>
</tr>
</tbody>
</table>

39. This semester, we learned how to translate arguments like

Socrates is a human.
All humans are mortal.
Thus, Socrates is mortal.

by using quantificational logic as follows:

\[ \text{Hs} \]
\[ \forall x (Hx \supset Mx) \]
Thus, Ms

But we already knew how to translate the argument given in English into sentential logic without introducing any symbols like \( \forall \) or \( \exists \). So why don’t we translate the argument into sentential logic instead of bothering with the more complicated quantificational translation?

If we did translate the argument into sentential logic, the argument would be invalid. But since the argument is plainly valid (by inspection), the translation would not capture the logic of the argument.