Logic—Sample Test B3 with Answers

NAME ____________________________________________________________

Identify the following arguments properly. If the argument is valid, mark it with a V. If it is invalid mark it with an I.

1. There is a mountain taller than every mountain in existence.  
   There is snow on the peak of the tallest mountain.
   VALID, the premise is a contradiction.

2. Jamie likes to eat Eskimo pies.  
   Thus, if Jamie doesn’t like to eat Eskimo pies, then she also dislikes pumpkin pies.
   VALID, It is of the form, A, Thus A ⊃ B.

3. The light will come on only if you flip the switch.  
   Thus, if you flip the switch, the light will come on.
   INVALID, It is of the form, A ⊃ B, Thus B ⊃ A.

4. If lightning strikes, then we won’t hear the thunder.  
   Thus, thunder is a kind of sound.
   VALID, the conclusion is a tautology.

5. Humans have stood on the surface of the moon.  
   Thus, if humans have never been near the surface of the moon, then they have stood on the surface of the moon.
   VALID, It is of the form, A, Thus B ⊃ A.

6. Suppose we add the symbol ‘⊗’ to our logic and we define its truth table as
   \[
   \begin{array}{ccc}
   \alpha & \beta & (\alpha \otimes \beta) \\
   T & T & F \\
   T & F & F \\
   F & T & F \\
   F & F & T \\
   \end{array}
   \]

   Define all four of the (standard) truth functions of our logic solely in terms of this new connective.
   \[
   \begin{align*}
   \neg \alpha &= (\alpha \otimes \alpha) \\
   (\alpha \land \beta) &= ((\alpha \otimes \alpha) \otimes (\beta \otimes \beta)) \\
   (\alpha \lor \beta) &= ((\alpha \otimes \beta) \otimes (\alpha \otimes \beta)) \\
   (\alpha \Rightarrow \beta) &= (\beta \otimes (\alpha \otimes \beta)) \otimes (\beta \otimes (\alpha \otimes \beta))
   \end{align*}
   \]
Translate the following sentences into the language of sentential logic using the given abbreviations. Remember that you do not need to worry about tense. “The food is good,” is equivalent to “The food will be good.”

F = “The food is good.”
G = “Guy makes potatoes.”
J = “Joe makes potatoes.”
H = “Henry cooks.”
P = “Paul cooks.”

7. “If Henry, but not Paul, cooks, then the food won’t be good.”
   \( (H \& \sim P) \supset \sim F \)

8. “Unless Guy makes potatoes, the food will be good.”
   \( \sim G \supset F \)

9. “Guy is makes potatoes if and only if Joe does too.”
   \( (G \supset J ) \& (J \supset G ) \)

10. “If Joe doesn’t make potatoes then the food will only be good if Guy makes them instead.”
    \( \sim J \supset (F \supset G) \)

11. “Unless Paul is doing the cooking, if the food turns out to be good, then it is Henry who is cooking.”
    \( \sim P \supset (G \supset H) \)

12. “If neither Paul nor Henry cooks, the food won’t be any good.”
    \( \sim (P \lor H) \supset \sim F \)

13. “In every case where the food is good, either Guy is making potatoes and Henry is cooking or Joe is making potatoes with Paul cooking.”
    \( F \supset ((G \& H) \lor (J \& P)) \)

14. “The only way Henry ever cooks is if Joe and Guy are making potatoes.”
    \( H \supset (J \& G) \)

15. “Barring some circumstance where Guy is making potatoes, Paul will do the cooking.”
    \( \sim G \supset P \)

16. “It is neither the case that Paul does the cooking if Henry does, nor is it the case that Guy makes potatoes if Joe does.”
    \( \sim ((H \supset P) \lor (J \supset G)) \)
Use the truth tree method to determine whether the set of sentences is consistent. Number all lines. Label all derived lines with the rule and the line from which they were derived. Answers should look just as in the notes. Label the set as consistent or inconsistent.

17. \{ \neg A \supset C, C \supset B, \neg (A \supset C) \}

18. \{ \neg (A \supset \neg \neg (C \& B)), \neg (B \lor D), (C \supset D) \}
Use the truth tree method to determine whether the argument is valid. Number all lines. Label all derived lines with the rule and the line from which they were derived. Answers should look just as in the notes. Label the argument as valid or invalid.

19. \[ D \supset \neg((A \land B) \land (B \lor C)) \]

\[ \frac{D}{\neg B \land A} \]

1. \[ D \supset \neg((A \land B) \land (B \lor C)) \]
2. \[ D \]
3. \[ \neg (\neg B \land A) \]
4. \[ \neg D \] \hspace{0.5cm} \neg((A \land B) \land (B \lor C)) \hspace{0.5cm} 1, \neg

5. \[ \neg (A \land B) \] \hspace{0.5cm} \neg(B \lor C) \hspace{0.5cm} 4, \neg \land
6. \[ \neg B \] \hspace{0.5cm} \neg C \hspace{0.5cm} 5, \neg \lor
7. \[ \neg A \] \hspace{0.5cm} \neg B \hspace{0.5cm} \neg C \hspace{0.5cm} 5, \neg \land
8. \[ \neg B \land A \] \hspace{0.5cm} \neg B \land A \hspace{0.5cm} \neg B \land A \hspace{0.5cm} 3, \neg \land

Invalid

20. \[ \neg (P \lor T) \]

\[ \frac{\neg P \supset (\neg R \lor T)}{\neg T} \]

1. \[ \neg (P \lor T) \]
2. \[ \neg P \supset (\neg R \lor T) \]
3. \[ \neg \neg T \]
4. \[ T \] \hspace{0.5cm} 3, \neg \neg
5. \[ \neg P \] \hspace{0.5cm} 1, \neg \lor
6. \[ \neg T \] \hspace{0.5cm} 1, \neg \lor

Valid
21. Using the given atomic sentences, translate the argument below into the language of propositional logic. Use the truth tree method to determine whether the argument is valid. Number all lines. Label all derived lines with the rule and the line from which they were derived. Complete the truth tree. Answers should look just as in the notes.

Yolanda will come if Elena does too.
Elena will avoid coming only if Rachel and Julie come.
Unless Yolanda fails to come, Rachel will come.
Either Julie or Elena will come.

Let Y = “Yolanda is coming.”
Let E = “Elena is coming.”
Let R = “Rachel is coming.”
Let J = “Julie is coming.”

\[
E \Rightarrow Y \\
\neg E \Rightarrow (R \& J) \\
\neg \neg Y \Rightarrow R \\
\neg \neg (J \lor E) \\
\neg J \lor E
\]

\[
\begin{align*}
1. & E \Rightarrow Y \\
2. & \neg E \Rightarrow (R \& J) \\
3. & \neg \neg Y \Rightarrow R \\
4. & \neg \neg (J \lor E) \\
5. & \neg J \\
6. & \neg E \\
7. & \neg \neg E \Rightarrow (R \& J) \\
8. & x \Rightarrow R \\
9. & x \Rightarrow J \\
\text{Valid}
\end{align*}
\]