Translate the following sentences into the language of sentential logic using the abbreviations given to you.

\[ G = \text{“Jose will play goalkeeper.”} \]
\[ J = \text{“Jose will play offense.”} \]
\[ D = \text{“Dirk will play defense.”} \]
\[ S = \text{“Kelli will show up.”} \]
\[ M = \text{“Kelli will play midfield.”} \]
\[ K = \text{“Kelli will play defense.”} \]
\[ O = \text{“Walter will play offense.”} \]
\[ W = \text{“Walter will play defense.”} \]
\[ Q = \text{“Pete will play midfield.”} \]
\[ P = \text{“Pete will play defense.”} \]

1. “If Jose plays goalkeeper, then Dirk will play defense.”
\[ G \supset D \]

2. “Unless Kelli doesn’t show up, Dirk won’t play defense.”
\[ \sim S \supset \sim D \]

3. “Jose isn’t going to play goalkeeper, but he and Walter will play offense instead.”
\[ \sim G \& (J \& O) \]

4. “Kelli will play midfield only if she shows up and Pete doesn’t play defense.”
\[ M \supset (S \& \sim P) \]
The lack of a comma before the ‘and’ disambiguates the scope of the sentence.

5. “Kelli and Pete will either both play midfielder together, or play defense together.”
\[ (M \& Q) \lor (K \& P) \]

6. “Both Dirk and Pete will not play defense, and neither will Kelli.”
\[ (\sim D \& \sim P) \& \sim K \]

7. “Unless Walter and Pete don’t both play defense, Jose won’t play goalkeeper.”
\[ \sim (W \& P) \supset \sim G \]

8. “Kelli will play either midfield or defense, unless she doesn’t show up.”
\[ \sim S \supset (M \lor K) \]

9. “Walter will play offense only if Jose doesn’t play offense or goalkeeper.”
\[ O \supset \sim (J \lor G) \]

10. “Kelli, Dirk, Pete and Walter are not all going to play defense.”
\[ \sim ((K \& D) \& (P \& W)) \]
11. Determine whether the following statements are true. If true, explain why (with a proof if possible); if false, find a counterexample:
   a. Any statement that implies a consistent statement is consistent.
      FALSE: \((A \& \neg A)\) is a statement that implies the consistent statement \(A\). Yet, \((A \& \neg A)\) is not consistent.
   b. Any statement implied by a consistent statement is consistent.
      TRUE: If a consistent statement \(P\) implied an inconsistent statement \(C\) (a contradiction), there would be an argument with the premise \(P\) and conclusion \(C\). Because \(P\) is consistent, there is a possible world where it is true, and that same world has \(C\) false because \(C\) is always false. This possible world constitutes a counterexample, contrary to the assumption that \(P\) implies \(C\). Because the assumption that a contingent \(P\) implies an inconsistent \(C\) led to a contradiction (an absurdity), there cannot be a contingent statement that implies a contradiction.

12. Suppose we add the symbol ‘≡’ to our logic and we define its truth table as

<table>
<thead>
<tr>
<th>(\alpha)</th>
<th>(\beta)</th>
<th>((\alpha \equiv \beta))</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
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</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

   What should the truth tree rule be for a statement of the form \((\alpha \equiv \beta)\)?

   Derive two branches, one with \(\alpha\) and \(\beta\), the other with \(\neg \alpha\) and \(\neg \beta\).

   Use the truth tree method to determine whether the set of sentences is consistent.
   Number all lines. Label all derived lines with the rule and the line from which they were derived. Answers should look just as in the notes. Label the set as consistent or inconsistent.

13. \(\{ (I \& \neg T) \& Z, \neg I \supset \neg Z, \neg (Z \& I) \} \)
14. \{ \sim \sim W, K \land (B \lor \sim E), E \lor \sim (R \lor \sim (\sim T \supset B)) \}
Use the truth tree method to determine whether the argument is valid. Number all lines. Label all derived lines with the rule and the line from which they were derived. Answers should look just as in the notes. Label the argument as valid or invalid.

15. \[
\begin{align*}
\text{J} & \quad \rightarrow \quad (B \rightarrow (C \rightarrow (D \rightarrow A)))
\end{align*}
\]
16. \[\neg Q \supset (A \& S)\]
\[\neg (\neg L \& R)\]
\[(R \& Q) \lor A\]
\[\neg (A \lor (L \& Q))\]
17. Using the given atomic sentences, translate the argument below into the language of propositional logic. Use the truth tree method to determine whether the argument is valid. Number all lines. Label all derived lines with the rule and the line from which they were derived. Complete the truth tree. Answers should look just as in the notes.

E = “Selma is enticed by your proposal.”
H = “Selma is hostile.”
I = “Selma is irritated.”
D = “Selma will be delighted.”

Either Selma is hostile and irritated or she is enticed by your proposal. Selma will be both delighted and not irritated if she is enticed by your proposal. Thus, Selma is enticed by your proposal if and only if she is irritated.

Either Selma is hostile and irritated or she is enticed by your proposal. Selma will be both delighted and not irritated if she is enticed by your proposal. Thus, Selma is enticed by your proposal if and only if she is irritated.