You must write the following two definitions in full grammatical sentences.

1. Define ‘counterexample’. (10 points)

2. Define ‘valid’. (20 points)

Identify the following arguments properly. If the argument is valid, mark it with a V. If it is invalid mark it with an I. (These problems are worth 2 points each.)

3. It is cloudy outside.
   The ground is wet.
   Thus, it has been raining.

4. He took the test.
   She took the test.
   Thus, both he and she took the test.

5. All Tech students are female.
   Only students that attend Tech can take this test.
   Thus, every student taking this test is female.

6. It is not the case that both Luke and Henry fell.
   Henry did not fell.
   Thus, Luke fell.

7. All teachers are geniuses.
   Scott is a genius.
   Thus, Scott is a teacher.
Translate the following sentences into the language of sentential logic using the abbreviations given to you. (These problems are worth 2 points each.)

P = “We played the game.”
G = “We won.”
W = “We played well.”
F = “We played fairly.”
Q = “We played quickly.”

8. If we didn’t win, then we didn’t play well.

9. We played quickly and well, but we did not win.

10. Either we played quickly or we played well, but we did not win.

11. Either we both played well and quickly or we did not win.

12. We played the game and played well, but we did not play quickly nor did we win.

13. If we did not play the game, then we did not play well, fairly, or quickly.

W = “Water is a solvent.”
A = “Acetone is a solvent.”
T = “The test came out positive.”
E = “Ethelene is a solvent.”

14. If the test came out positive, then acetone and ethelene are solvents.

15. Acetone is a solvent only if water itself is a solvent.

16. Unless the test came out positive, acetone is definitely not a solvent.

17. Ethelene and water are not solvents, and neither is acetone.

18. Ethelene is a solvent if and only if water is not a solvent.

19. Water is not a solvent, unless the test did not come out positive.
20. Either water is a solvent if ethelene is, or the test did not come out positive.

21. If the test came out positive, then if water is not a solvent, neither is acetone.

Label each of the following sequences of symbols with a ‘Wff’ if and only if it is a well-formed formula and with an ‘X’ if and only if it is not a well-formed formula. (1 point each)

22. \((T \& \sim\sim E) \lor J\)

23. \(\sim\sim (K \& \sim K) \supset J\)

24. \((T \sim (E \lor L))\)

25. \((Y \lor (\sim\sim E \& \sim R) \& \sim J)\)

Construct truth tables to test whether these arguments are valid or invalid. In the case of an invalid argument, indicate the row or rows that show that the argument is invalid by one of them. (These problems are worth 3 points each.)

26. \(~B \& A\)

\(~B \supset C\)

\(C \supset A\)

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<th>(C &amp; A)</th>
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Valid or invalid? ________________________
(If it is invalid, circle any row that proves that it is invalid.)
27.  \(~(E \lor \sim D)\)  \\
\(D \lor \sim E\)  \\
\(\sim E\)  \\
\(E\)  \\

Valid or invalid? ________________________  \\
(If it is invalid, circle any row that proves that it is invalid.)

28.  \(~(R \supset M) \supset N\)  \\
\(~(M \& R)\)  \\
\(N\)  \\

Valid or invalid? ________________________  \\
(If it is invalid, circle any row that proves that it is invalid.)

Test whether each of these sentences is a tautology, a contradiction, or a contingent sentence by constructing their truth tables. (These problems are worth 3 points each.)

29.  \((A \& B) \lor (\sim A \& \sim B)\)  \\

Tautology, contradiction, or contingent? ________________________
30. \( (R \supset C) \lor (R \& \sim C) \)

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<th>( C )</th>
<th>( R \lor C )</th>
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Tautology, contradiction, or contingent? ________________________

31. Test the following pair of sentences for logical equivalence by constructing the truth tables. Label whether they are equivalent or inequivalent. If they are inequivalent, circle any row that proves that they are inequivalent. (3 points)

\( (\sim G \& \sim H) \supset G \)
\( (G \supset H) \lor G \)

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<thead>
<tr>
<th>( G )</th>
<th>( H )</th>
<th>( \sim G &amp; \sim H )</th>
<th>( \supset )</th>
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Equivalent or inequivalent? ____________________________
(If they are inequivalent, circle any row that proves that they are inequivalent.)

32. Test the following pair of sentences for consistency by constructing the truth tables. Label whether they are consistent or inconsistent. If they are consistent, circle any row that proves that they are consistent. (3 points)

\( (A \lor \sim B) \)
\( \sim (\sim A \lor B) \)

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<tr>
<th>( A )</th>
<th>( B )</th>
<th>( \sim A )</th>
<th>( \lor )</th>
<th>( B )</th>
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Consistent or inconsistent? ____________________________
(If it is consistent, circle any row that proves that it is consistent.)
Either Alice or Bob ate some of the pizza, and either Cindy or Alice did.
If Alice ate some of the pizza then neither Cindy nor Bob ate some of the pizza.
Unless both Cindy and Alice failed to eat some of the pizza, Bob ate some.
Therefore, Cindy ate no pizza.

A = “Alice ate some of the pizza.”
B = “Bob ate some of the pizza.”
C = “Cindy ate some of the pizza.”

Translate the argument into symbols here:

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Fill out the truth table here.

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Valid or invalid? ________________
(If it is invalid, circle any row that proves that it is invalid.)