1 Translations

When to use the parentheses: We only use the parentheses for formulas or subformulas that are conjunctions, disjunctions, or conditionals. And we can omit the outermost pair of parentheses. You can refer to p. 67 of the notes for the details. No points were deducted for writing a formula that is not well-formed, as long as the formula you wrote doesn’t have more than one readings.

Examples of incorrect uses of parentheses:
\[ \exists x(Sx \lor Px); \forall x(\sim \sim Fx); (\forall xSx) \exists x \sim (Sx) \]

Examples of correct uses of parentheses:
\[ \exists x(Sx \supset Px); \forall x(Px \& \sim \sim Fx); \exists y(\exists x Sx \& Py) \]

Someone; somewhere; anyone; anywhere: Remember that someone is F; somewhere is F; anyone is F; anywhere is F should be translated as \[ \exists x(Px \& Fx) \]; \[ \exists x(\exists x Sx \& Fx) \]; \[ \forall x(Px \supset Fx) \]; \[ \forall x(\exists x \supset Fx) \] respectively.

Unless: Remember that ‘unless α, β’ or ‘β, unless α’ should be translated as \[ \sim \sim α \supset β \] or simply as \[ α \lor β \].

(5) Trent is going to the mall, but there aren’t any toys he’s shopping for.
   Answer: Gtm \& \sim \exists x(Tx \& Stx)

   Incorrect translation

   (i) Gtm \& \exists x(Stx \& \sim Tx). This translation says that Trent shops something that is not a toy, but (5) doesn’t say that Trent shops for something.

(6) If Rachel is going anywhere, the mall will be among those places where she goes.
   Answer: \exists x((Lx \& Grx) \supset Grm)

   Incorrect translation

   (i) \exists x((Lx \& Grx) \supset Grm). The translation says: there is an object x such that if x is a location that Rachel goes to, then Rachel goes to the mall. Notice that this translation asserts the existence of an object which has a certain property. But the sentence (6) doesn’t assert that. (For (6) to be true, the antecedent of (6) can be false).

   Alternative translation

   (i) \forall x((Lx \& Grx) \supset Grm). It is logically equivalent to \exists x((Lx \& Grx) \supset Grm).

(8) Unless Rachel goes to the mall or downtown, she will not shop for anything.
   Answer: \sim ((Grm \lor Grd) \supset \sim \exists xSr)

   Alternative translation

   (i) \forall x((Grm \lor Grd) \supset \sim Sr). It is locally equivalent to \sim ((Grm \lor Grd) \supset \forall x \sim Sr).

(9) There is somewhere Trent will go if he’s broke.
   Answer: Bt \supset \exists x(Lx \& Gt)
Alternative translations

(i) $\exists x(Lx \& (Bt \supset Gtx))$. (9) can be interpreted as asserting the existence of a place where Trent will go if he's broke.

(ii) $\exists x(Bt \supset (Gtx \& Lx))$ which is logically equivalent to $Bt \supset \exists x(Gtx \& Lx)$.

(10) If someone goes to the mall, they will also go downtown.
Answer: $\forall x[(Px \& Gxm) \supset Gxd]$

Alternative translation

(ii) $\exists x(Bt \supset (Gtx \& Lx))$ which is logically equivalent to $Bt \supset \exists x(Gtx \& Lx)$.

Incorrect translations

(i) $\exists x((Px \& Gxm) \supset Gxd)$. ‘someone’ in (10) is better understood to mean ‘anyone’. Also, this translation asserts the existence of an object (which is such that if it is a person and goes to the mall, it goes downtown). But (10) doesn’t assert that.
Since $\forall x((Px \& Gxm) \supset Gxd)$ doesn’t imply $\exists x((Px \& Gxm) \supset Gxd)$, it is a better translation.

(ii) $\exists x(Px \& Gxm) \supset Gxd$. Notice that the occurrences of ‘$x$’ in ‘$Px$’ and in ‘$Gxm$’ are inside the scope of the existential quantifier; they are said to be bound by the quantifier. The occurrence of ‘$x$’ in ‘Gxd’ is not bound by the quantifier, so it is unrelated to the other two occurrences of ‘$x$’.

(iii) $\exists x(Px \& Gxm) \supset \exists x(Px \& Gxd)$. It says if someone goes to the mall, then someone goes downtown. It is incorrect, because it doesn’t say that the people who go to the mall are the very same people who go downtown.

(iv) $\forall x((Px \supset Gxm) \supset Gxd)$. It is not logically equivalent to $\forall x(Px \supset (Gxm \supset Gxd))$ or to $\forall x((Px \& Gxm) \supset Gxd)$.

(v) $\forall x(Px \& Gxm \supset Gxd)$. It is not logically equivalent to $\forall x(Px \supset (Gxm \supset Gxd))$ or to $\forall x((Px \& Gxm) \supset Gxd)$.

(11) There isn’t any candy if Rachel is broke.
Answer: $Br \supset \forall x \neg Cx$

Alternative translation

(i) $\forall x(Br \supset \neg Cx)$. It is logically equivalent to $Br \supset \forall x \neg Cx$.

(13) If anyone goes downtown, someone will go to the mall.
Answer: $\exists x(Px \& Gxd) \supset \exists x(Px \& Gxm)$

Alternative translation

(i) $\forall x[(Px \& Gxd) \supset \exists y(Py \& Gym)]$, which is logically equivalent to $\exists x(Px \& Gxd) \supset \exists x(Px \& Gxm)$

Incorrect translations

(i) $\forall x(Px \supset Gxd) \supset \exists x(Py \& Gxm)$. ‘anyone’ in (13) is better understood to mean ‘someone’.
(ii) $\exists x[(Px \& Gxd) \supset \exists y(Py \& Gym)]$. It is not logically equivalent to $\exists x(Px \& Gxd) \supset \exists x(Px \& Gxm)$

(iii) $\exists x[(Px \& Gxd) \supset Gxm]$. In order for (13) to be true, the people who go downtown need not be the very same people who go to the mall.

**Complicated but incorrect translation**

(i) $\forall x\exists y[(Px \& Py) \& Gxd \supset Gym]$. If you want to see why it is incorrect, consider the following situation in which (13) and the translation have different truth values: suppose world X contains three objects only: Ann, the downtown, and the mall. They are denoted by constants $a, d$, and $m$ respectively. Ann goes to the downtown, but she doesn’t go to the mall. So, (13) is false in world X. However, the translation says of each object $x$ in world X that $\exists y[(Px \& Py) \& Gxd \supset Gym]$. i.e. The following three formulas are true:
   a. $\exists y[(Pa \& Py) \& Gad \supset Gym]$
   b. $\exists y[(Pd \& Py) \& Gdd \supset Gym]$
   c. $\exists y[(Pm \& Py) \& Gmd \supset Gym]$
   In fact, they are all true, because world X contains objects that are not persons. So, the translation is true in world X.

(14) Among all people, only the broke ones go downtown.
   Answer: $\forall x((Px \& Gxd) \supset Bx)$

**Complicated but incorrect translation**

$\forall x\forall y[(Px \& Bx) \& (Py \& \sim By) \supset (Gxd \& \sim Gyd)]$. It says something like this: any pair of persons $\langle x, y \rangle$ is such that if $x$ is broke and $y$ is not, then $x$ goes downtown and $y$ doesn’t. To see why this translation is incorrect, consider the following situation in which (14) and the translation have different truth values: suppose there are only three people in world X: Amy, Bob, and Chris. Only Amy and Bob are broke. Only Amy goes downtown. So, (14) is true in world X. But the translation says of $\langle Bob, Chris \rangle$ that if Bob is broke and Chris isn’t, then Bob goes downtown and Chris doesn’t. But this is not true, because, in world X, even though Bob is broke and Chris isn’t, Bob doesn’t go downtown.

(16) The only things Rachel will possibly shop for are toys and candy.
   Answer: $\forall x(Srx \supset (Tx \vee Cx))$

**Incorrect translations**

(i) $\forall x(Srx \supset (Cx \& Tx))$. It says that anything that Rachel shops for is both a toy and a candy.

(ii) $\exists x Srx \supset (Cx \vee Tx)$. The last two occurrences of ‘$x$’ are unrelated to the occurrence of ‘$x$’ that is bound by the existential quantifier.

**Complicated but incorrect translation**

(i) $\exists x\exists y[(Srx \vee Sry) \supset (Cx \vee Ty)]$. Consider the following situation in which the translation and (16) have different truth values. Suppose Rachel shops for a candy and a mango (not a toy). They are denoted by constants $c$ and $m$ respectively. The following formula is true: $(Src \vee Srm) \supset (Cc \vee Tm)$ (since ‘$Src’ and ‘$Cc’ are true). So it follows that the translation is true. But (16) is clearly false, since Rachel shops for a mango (which is not a toy).
2 Truth Trees

17. (i) We should always instantiate the existentially quantified wffs before instantiating the universally quantified wffs.

(ii) Since constants 'a' and 'b' have already appeared on the truth trees, we should instantiate ‘∃xPax’ on a new constant.

(iii) We cannot use the universal (existential) quantifier rule on ∃xPax&∀x ~ Pxb to get

\[ \exists x Pax \& \sim Pab \]

or

\[ Pac \& \forall x \sim Pxb \]

We can only use the rule when the universally (existentially) quantified wff stands on its own.

(iv) We shouldn’t instantiate a quantified wff on a variable (i.e. a lower case letters from u to z). We always instantiate it on a constant (i.e. the other lower case letters).

(v) The notion of consistency applies to a set of sentence(s). So, we can ask whether a sentence (or a set that contains only one sentence) is consistent or not.

18. (i) We are not supposed to use the existential quantifier rule and the universal quantifier rule at the same time like this:

\[ \forall y (Py \supset \exists z Gz) \]
\[ Pa \supset Gb \]

(ii) The following application of the existential quantifier rule is also incorrect:

\[ \forall y (Py \supset \exists z Gz) \]
\[ \forall y (Py \supset Gc) \]

(iii) When we instantiate the universally quantified wffs, we should instantiate them on constants that have appeared in the truth tree, because our goal is to see whether there are contractions.

20. (i) General strategy: if you have a conjunction like \( \forall x (Rx \supset Ax) \& \forall x (Mx \supset Ax) \), you should use the conjunction rule and break it down into two wffs first. In general, if you have a conjunction, break it down first.

(ii) General strategy: if you have negated quantified wffs (e.g. \( \sim \exists x Fx; \sim \forall x (Gx \& Hx) \)), you should use the rule QE on them before your tree branches out.

(iii) ‘\( \forall x [(Rx \supset Ax) \& (Mx \supset Ax)] \)’ is not a conjunction. So, you cannot use the rule & on it.
(iv) We cannot use the rule $\forall$ on $\forall x (Rx \supset Ax) \& \forall x (Mx \supset Ax)$ to get $\forall a (Ra \supset Ax) \& (Ma \supset Aa)$ in one step.