NAME __________________________________________________________

Translate the following sentences into the language of sentential logic using the abbreviations given to you. Only use the standard symbols, ~, &, v, ⊃. (3 points each.)

\[ B = \text{“Theresa will ride her bicycle.”} \]
\[ F = \text{“Theresa’s car is functioning correctly.”} \]
\[ A = \text{“Yesterday, Theresa had an accident.”} \]
\[ W = \text{“Theresa is doing well.”} \]
\[ E = \text{“The radiator exploded.”} \]
\[ M = \text{“Theresa maintained her car properly.”} \]

1. Theresa maintained her car properly, but it is nevertheless malfunctioning, and in any case, she will be riding her bicycle.
   \[ M \& (\neg F \& B) \]

2. Even if Theresa’s car is functioning correctly, she is still going to ride her bicycle.
   \[ B \]

3. Theresa is doing well only if it turned out that no accident befell her yesterday.
   \[ W \supset \neg A \]

4. Theresa will ride her bicycle if and only if her car is malfunctioning.
   \[(B \supset \neg F) \& (\neg F \supset B)\]

5. Theresa is not doing well regardless of whether she had an accident yesterday or not.
   \[ \neg W \quad \text{or} \quad (A \supset \neg W) \& (\neg A \supset \neg W) \]

6. If the radiator did not explode and Theresa maintained her car properly, then her car is not malfunctioning.
   \[(\neg E \& M) \supset \neg \neg F \]

7. Whenever Theresa is riding her bicycle, either her car is malfunctioning or she is doing well.
   \[ B \supset (\neg F \lor W) \]

8. It is untrue that if Theresa had an accident yesterday, she is not doing well.
   \[ \neg (A \supset \neg W) \]

9. Unless Theresa failed to maintain her car properly, she will not ride her bicycle.
   \[ \neg \neg M \supset \neg B \]

10. Either the radiator exploded or Theresa’s car is malfunctioning; either way, Theresa had an accident yesterday.
    \[(E \lor \neg F) \& A \]
11. Consider the following formulation of the hypothetical syllogism using Adams’ probability conditional. Prove that it is probabilistically invalid by drawing a diagram depicting regions whose areas correspond to the probability of the conditional. (Recall that an argument is probabilistically invalid iff it has a probabilistic counterexample. A probabilistic counterexample is a possibility where the probability of all the premises is very high (approaching 1) while the probability of the conclusion is very low.) (5 points)

\[ A \Rightarrow B \\
B \Rightarrow C \\
A \Rightarrow C \]

Consider the following formulation of the hypothetical syllogism using the material conditional. Prove (any way you like) that it is valid in the ordinary truth-preserving sense. (5 points)

\[ A \supset B \\
B \supset C \\
A \supset C \]

1. \[ A \supset B \]
2. \[ B \supset C \]
3. \[ \neg (A \supset C) \]
4. \[ A \] 3, \neg C
5. \[ \neg C \] 3, \neg C
6. \[ \neg A, B \] 1, C
7. \[ \neg B, C \] 2, C

Valid
Logic—Second Exam

Use the truth tree method to determine whether the set of sentences is consistent. Complete the truth tree. Number all lines. Label all derived lines with the rule and the line from which they were derived. Answers should look just as in the notes. Label the set as consistent or inconsistent. (10 points each.)

12. \{ \overline{((F \land \overline{A}) \lor (B \land \overline{F})), A \land (B \supset F)} \}

13. \{ K \supset (\overline{R \supset S}), \overline{\overline{\overline{R} \lor S}}, \overline{\overline{K} \supset K} \}
Use the truth tree method to determine whether the argument is valid. Complete the truth tree. Number all lines. Label all derived lines with the rule and the line from which they were derived. Answers should look just as in the notes. Label the argument as valid or invalid. (10 points each.)

14. \(~E \supset \sim W\)
   \(~(E \lor \sim D)\)
   \(~D \lor \sim W\)
   \(~(H \land D)\)

15. \(G \lor B\)
   \(G \supset Q\)
   \(B \supset S\)
   \(S \lor Q\)
16. Using the given atomic sentences, translate the argument below into the language of propositional logic. Use the truth tree method to determine whether the argument is valid. Number all lines. Label all derived lines with the rule and the line from which they were derived. Complete the truth tree. Answers should look just as in the notes. (3 points for each translation, 8 points for the truth tree.)

Unless Steve has to work, he will repair the fence.
Even if he has to work, Steve will repair the gate.
Steve is not going to repair both the gate and the fence.
Steve is going to take the truck only if he has to work.

Let W = “Steve has to work.”
Let F = “Steve will repair the fence.”
Let G = “Steve will repair the gate.”
Let T = “Steve will take the truck.”

\[
\neg(F \land G) \\
\neg W \supset F \\
G \\
T \supset W
\]

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<tr>
<th>Line</th>
<th>Formula</th>
<th>Rule</th>
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<tbody>
<tr>
<td>1</td>
<td>\neg(F \land G)</td>
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<td>2</td>
<td>\neg W \supset F</td>
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<td>3</td>
<td>G</td>
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<td>\neg(T \supset W)</td>
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<td>8</td>
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<tr>
<td>9</td>
<td>\neg W</td>
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Valid